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Entangled quantum clocks for measuring proper-time difference

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Abstract. We report that entangled pairs of quantum clocks (non-degenerate quantum bits) can be used as a specialized detector for precisely measuring difference of proper-times that each constituent quantum clock experiences. We describe why the proposed scheme would be more precise in the measurement of proper-time difference than a scheme of two-separate-quantum-clocks. We consider possibilities that the proposed scheme can be used in precision test of the relativity theory.

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It is quantum entanglement that led to the historical controversy over Einstein-Podolsky-Rosen experiment [1] and then led to the Bell's inequality [2] that explicitly revealed non-local nature of quantum mechanics. On the other hand, entanglement is the key ingredient in quantum information processing: for example, the speedup in quantum computation [3] is obtained through the parallel quantum operations on massively superposed states which are entangled in general. Recently, several new protocols using quantum entanglement that have advantages over its classical counterparts were proposed-entanglement enhanced frequency measurement [4], quantum lithography [5,6], quantum clock synchronization based on shared prior entanglement [7–11], efficient quantum clock-transport scheme [12], and quantum enhanced positioning [13].

In this paper, we propose a new application of the entangled pairs of quantum clocks (non-degenerate quantum bits) the specialized detector that precisely measures difference of proper-times that each quantum clock experiences. The proposed scheme is expected to be more precise in measuring the proper-time difference than a scheme where two separate quantum clocks are employed. In this scheme, quantum clocks need to be accelerated for some time-intervals and the acceleration's effects on quantum clocks might be non-negligible. Thus appropriate handling of the effects are necessary. We suggest a solution and a utilization of this case. Then we consider using the proposed scheme in the precision test of relativistic time-dilation effects.

Let us assume that we have entangled pair of quantum clocks in the state

$$|\Psi^-\rangle = |0\rangle_A|1\rangle_B - |1\rangle_A|0\rangle_B, \quad (1)$$

where A and B respectively corresponds to each quantum clock whose proper-time difference will be compared. (The normalization factor is omitted throughout this paper.) We also assume that Hamiltonian $\mathbf{H}_\alpha$ for two mutually orthogonal states of a quantum clock, $|0\rangle_\alpha$ and $|1\rangle_\alpha$ ($\alpha = A, B$) is given by

$$\mathbf{H}_\alpha = E_\alpha \sigma_z, \quad (2)$$

where σ_i ($i = x, y, z$) is the Pauli operators. The time evolution of each quantum clock is in general given by the unitary operation

$$U_\alpha(t)|0\rangle_\alpha = \mathrm{e}^{\mathrm{i}E_\alpha t}|0\rangle_\alpha, \quad U_\alpha(t)|1\rangle_\alpha = \mathrm{e}^{-\mathrm{i}E_\alpha t}|1\rangle_\alpha, \quad (3)$$

where $\hbar$ is set to be one. When two clocks follow different space-time trajectories, the time for each clock is given by it's own proper-time. First let us consider the case $E_A = E_B = E$. (We will later consider a general case where E_α's are time-dependent and thus are not the same.) After proper-times t_A and t_B have elapsed for A and B quantum

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clocks, respectively, the initial state of the quantum clocks in equation (1) becomes

$$\begin{aligned} &U_A(t_A)U_B(t_B)|\Psi^-\rangle \\ &\quad = U_A(t_A)(\mathrm{e}^{-\mathrm{i}Et_B}|0\rangle_A|1\rangle_B - \mathrm{e}^{\mathrm{i}Et_B}|1\rangle_A|0\rangle_B), \\ &\quad = \mathrm{e}^{-\mathrm{i}Et_B}\mathrm{e}^{\mathrm{i}Et_A}|0\rangle_A|1\rangle_B - \mathrm{e}^{\mathrm{i}Et_B}\mathrm{e}^{-\mathrm{i}Et_A}|1\rangle_A|0\rangle_B, \\ &\quad = \mathrm{e}^{\mathrm{i}E\Delta t}|0\rangle_A|1\rangle_B - \mathrm{e}^{-\mathrm{i}E\Delta t}|1\rangle_A|0\rangle_B, \qquad (4) \end{aligned}$$

where $\Delta t = t_A - t_B$. In the proposed scheme we initially prepare quantum clock pairs in the state $|\psi^-\rangle$ at a single site. We let each quantum clock (labeled by A or B) departs and follows its own space-time trajectory and gather them again. Then we perform some (collective) measurement on the quantum clocks and get information about the proper-time difference Δt. (We do not consider the case where the terms differ by $2n\pi$, n is integer.) As we see, the proper-time difference Δt contributes to the relative phase of the quantum clock pair. Thus we can determine Δt by measuring the relative phase. In other words, *the difference Δt of the proper-time that each quantum clock experiences since they departed, is accumulatedly recorded in the relative phase of the non-degenerate quantum clock pair in equation (4), which can be read out by collectively measuring the quantum clocks at a single site.* In the quantum clock synchronization [7], it is required that the state remains in the initial one when each clock has arrived at its own location. Namely a condition that $t_A = t_B$ should be satisfied. This condition can be fulfilled by slow transportation of quantum clocks. In this case, the relativistic effect is something to be suppressed by a careful manipulation (slow transportation) of quantum clocks. In contrast, the proposed scheme utilizes the (relative) phase rotation of the state in equation (4) when we measure the relativistic time-dilation effect.

In the following, we explain why the proposed scheme would be more precise in measuring the proper-time difference than the scheme of two-separate-quantum-clocks. In the latter, proper-times of two separate quantum clock which have traveled through different space-time trajectories are compared to estimate the difference between them.

Roughly speaking, in the proposed scheme the (relative) phase corresponding to the proper-time difference become stationary while measurement is done. Thus the proper-time difference can be more accurately measured in the proposed scheme.

Let us consider simple measurement models and then, using these, discuss on the advantage of the entangled scheme.

In separate quantum clocks scheme, quantum clocks are initially prepared in the state $|\bar{0}\rangle = |0\rangle + |1\rangle$. (In this notation, $|\bar{1}\rangle = |0\rangle - |1\rangle$.) In order to measure the phase, we perform, for example, a measurement $\hat{S}_x$ composed of two projection operators $|\bar{0}\rangle\langle\bar{0}|$ and $|\bar{1}\rangle\langle\bar{1}|$. The measurement $\hat{S}_x$ on αth quantum clock can be done by applying the following interaction Hamiltonians $\mathbf{H}^I_\alpha$ between each quantum clock and an ancillary quantum bit for a time width δt [14–16]

$$\mathbf{H}^I_A = \sigma_x \otimes I \otimes F, \qquad \mathbf{H}^I_B = I \otimes \sigma_x \otimes F, \qquad (5)$$

where I is the identity operator and F is a certain operator that acts on ancillary quantum bit. Here the time width δt is inevitably finite because it describes real physical processes. Let us consider A-quantum clock. (The same thing can be said for B-quantum clock.) The total Hamiltonian $\mathbf{H}^T_A$ can be written as

$$\begin{aligned} \mathbf{H}^T_A &= \mathbf{H}_A + g(t)\mathbf{H}^I_A \\ &= E\sigma_z \otimes I \otimes I + g(t)\sigma_x \otimes I \otimes F, \qquad (6) \end{aligned}$$

where $g(t)$ is a Gaussian-like function that is peaked at the time when measurement is performed and whose half-width is δt. During the measurement, the prepared quantum clock rapidly rotates between $|\bar{0}\rangle$ and $|\bar{1}\rangle$ due to its own Hamiltonian $\mathbf{H}_A$. Since $[\mathbf{H}_A,\ \mathbf{H}^I_A] \neq 0$ ($[C,D] = CD - DC$) and $\delta t \neq 0$, the measurement result is inevitably affected by the evolution due to $\mathbf{H}_A$. (To suppress this effect, it is assumed that either $\mathbf{H}_A = 0$ or $\delta t \to 0$ in many cases [14–16].) Namely, during the measurement interval δt, the phase to be measured is rotated $2\pi\delta t/T (T = \pi/E)$. Thus, if the measurement time-width δt is non-negligible comparing with the period of rotation T, the result of the measurement would be an average of phases of all states in which the prepared quantum clock stays during a full rotation. In this case, therefore, the measurement would fail or at least be largely uncertain, if $\delta t > T$. Now let us consider the proposed scheme. Here the pair of quantum clocks are prepared in the state $|\Psi^-\rangle$ in equation (1). In order to measure the relative phase in equation (4) later, we perform, for example, a measurement with the following interaction Hamiltonian

$$\mathbf{H}^I = \boldsymbol{\sigma}^2_T \otimes F, \qquad (7)$$

where $\boldsymbol{\sigma}_T = \boldsymbol{\sigma} \otimes I + I \otimes \boldsymbol{\sigma}$ and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$. This corresponds to total-spin measurement [17] in the case where the quantum clocks are spin-1/2 states. Similarly to above separate-case, the total Hamiltonian $\mathbf{H}^T$ is given by

$$\mathbf{H}^T = \mathbf{H}_A + \mathbf{H}_B + g(t)\mathbf{H}^I. \qquad (8)$$

However, since $[\mathbf{H}^I, \mathbf{H}_A + \mathbf{H}_B] = 0$ here, we can safely measure the quantity corresponding to $\boldsymbol{\sigma}^2_T$ [14,15]. Then let us decompose the state in equation (4) as

$$\mathrm{e}^{\mathrm{i}E\Delta t}|0\rangle_A|1\rangle_B - \mathrm{e}^{-\mathrm{i}E\Delta t}|1\rangle_A|0\rangle_B = \cos(E\Delta t)|\Psi^-\rangle + \mathrm{i}\sin(E\Delta t)|\Psi^+\rangle, \quad (9)$$

where $|\Psi^-\rangle$ and $|\Psi^+\rangle$ are eigenstates of $\boldsymbol{\sigma}^2_T$ with eigenvalues 0 and 1, respectively. ($|\Psi^+\rangle = |0\rangle_A|1\rangle_B + |1\rangle_A|0\rangle_B$.) However, the relative phase $E\Delta t$ does not evolve at the measurement stage and thus finiteness of δt does not matter. Therefore, by measuring $\boldsymbol{\sigma}^2_T$ with $\mathbf{H}^I$ in equation (7), for example, we can obtain coefficients in equation (9) and then calculate Δt.

Now let us continue to discuss on advantage of the proposed scheme. It is clear that accuracy of the two-separate-clocks scheme is limited by the uncertainty of each separate clock. That is, the proper-time difference cannot be measured more accurately than the uncertainty of each clock's time. (Here we assume the period $T = \pi/E$ of quantum clock's phase rotation is a constant, which is equivalent to assuming complete shielding of quantum clocks from environments. Incompleteness of the shielding might be the limiting factor for quantum clocks in some cases. In this case, phase-uncertainty improvement by the proposed scheme would not be of much importance. Thus what we consider is the case where such complete-shielding problem is overcome by certain methods. Similar thing can be said to the efficient quantum clock transport scheme [12] which improves phase-uncertainty. However, even in this case, the phase uncertainty would limit the accuracy of quantum clocks.)

The accuracy of a quantum clock is roughly proportional to the product of the period T of phase rotation and the uncertainty in the phase measurement $\delta\phi$. The uncertainty of the phase may be due to the inherent statistical behavior of quantum states (*i.e.*, the results of the phase-measurement form a statistical distribution given by quantum mechanical formula) and inherent finite time width such as δt of the function $g(t)$ in equations (6, 8) involved with phase-measurement. If we employ many quantum clocks, we can reduce the phase uncertainty; roughly 2^{2n} number of quantum clocks allow us to estimate n bits of the phase [12]. When the number of quantum clocks is given, one may further improve the accuracy by decreasing T *i.e.* by increasing the speed of phase rotation. (We can consider improvement of accuracy by other method, namely by optimizing the initial states [18].) However, this method has its own limitation as the following. The faster a phase rotates the larger the phase uncertainty would become, since the phase makes wider angle of rotation during the measurement: in real experiment our measurement-results for the phase would inevitably correspond to the phases during the inherent finite time width such as δt of the function $g(t)$ in equations (6, 8), not that of an instance. Thus the measurement-results for phases make a broader statistical distribution than in the case where δt is zero, thus increasing the phase uncertainty for a given number of quantum clocks. In particular, the δt's broadening effect would be considerable when T become comparable with δt. Moreover when T become smaller than δt, due to cyclic property of phase, the phase uncertainty would be rapidly maximized so that it would become impractical to determine the phase. (Therefore an optimal accuracy of quantum clocks would be obtained by employing quantum bit systems with a certain T of intermediate value.)

In contrast, in the proposed scheme the relative phase in equation (4) to be measured does not rotate while measurements for the relative phase, for example, the σ_T^2 measurement, are being performed. Thus the inherent finite time width δt involved with phase-measurement does not matter in the proposed scheme. On the other hand, we can see that accuracy of the proposed scheme is given by a product of each separate quantum clock's period T of phase rotation and uncertainty in the relative phase $\delta\phi$. (Here note that T is not the period of the relative phase's rotation but fast rotation of each quantum clock's phase.) Thus we can improve the accuracy, by decreasing T as we like without increasing $\delta\phi$ in the proposed scheme.

Now let us consider quantum clocks whose phase rotation can be turned on or off, as we like by some operation. For example, spin precession of particles by applied magnetic field can used as quantum clocks. Here we can make the clocks turned on (off) by applying nonzero (zero) magnetic field. In this case, accuracy of the two-separate-clocks scheme also would not be limited by the δt, since we may turn off both clocks when we are measuring them. However, in this case the magnetic field instead must be precisely controlled to the level of required accuracy of the scheme, which would be a much more difficult task than attaining the required accuracy with two naturally given energy eigenstates as in ordinary quantum clocks scheme.

The assumption that environmental effects can be efficiently removed is crucial for the success of the proposed scheme. One may ask that if such efficient shielding is possible or accurate quantum clocks can be obtained then why we need the entangled quantum clocks scheme. However, as noted above, efficient shielding would not directly guarantee accurate quantum clocks, due to uncertainty in phase measurement. Overcoming the phase uncertainty would become particularly important in precise measurement of the difference of proper-times. The proposed scheme is advantageous in that it is not limited by the inherent time width δt involved with phase-measurement, in overcoming the phase uncertainty.

Let us now consider the general case where E_α's are time-dependent and thus are not the same. E_α may be time-dependent due to either interaction with environments or acceleration that each quantum clock suffers during the round trip in space-time. As previously, here we assume that the environmental effects can be removed by shielding the quantum clocks from environment. It is not well known yet how quantum clocks are perturbed by acceleration except for the fact that the effect would be very small [21]. The condition that $E_A = E_B = E$ we assumed previously also implies that acceleration effects can be removed by some methods, *e.g.*, careful choice of the system to be used as quantum clocks or simply making the acceleration very small. Now we consider the case where the acceleration effects on the quantum clock's time evolution is non-negligible. The phase difference $\Delta\phi = \int_0^{t_A} E_A(t'_A)\mathrm{d}t'_A - \int_0^{t_B} E_B(t'_B)\mathrm{d}t'_B$ that we would measure in the proposed scheme is the combined results of the relativistic and the acceleration effects, which cannot be discriminated from each other in the measurement result. Nevertheless, we can still utilize the proposed scheme for measuring the proper-time difference by making each quantum clock to experience the same acceleration effect while following different path in space-time. For example, we can consider the following case almost similar to the twin-paradox experiment [19]. One (the other) party makes a short (long) trip. However their accelerating sections in the space-time trajectory are the same with each other. In this case the acceleration effect cancels with each other and thus we can estimate pure relativistic time-dilation effect from the measurement result. Let us consider another example, gravitational time-dilation [20, 21] where we can also make the acceleration effects to cancel with each other. First prepare the two parties of entangled quantum clocks at one site in constant gravitational field. Then lift both of them to a higher place and then bring down one party to the original place. After waiting for a long time, bring down the other party with the same magnitude of acceleration and velocity as the first one. Then by measuring the phase difference we can estimate the gravitational time dilation. On the other hand, we can make use of the proposed scheme for measuring the acceleration effect. Let one party to be at rest and another party to make a trip with some acceleration, as we do in the original twin-paradox experiment [19]. Measure the phase difference $\Delta\phi$ and calculate each party's proper-time using formula of special relativity. The difference between them is the acceleration effect.

The precision of the entangled quantum clocks scheme is estimated to be order of period T, assuming $\delta\phi \sim 1$. The period T of hyperfine transition is of order of 10^{-10} s. However, in the proposed scheme a system with more rapidly rotating phase can be employed since the phase become stationary while it is being measured, as noted before. By choosing some quantum clocks whose energy difference E_α is of an order of one electron volt, one can obtain $T \sim 10^{-14}$ s. However, the greater energy difference E_α becomes, the more probable the higher energy state makes a spontaneous transition to the lower one in general. This problem may be avoided by adopting some metastable states as quantum clocks, although this problem would limit the accuracy of the proposed scheme.

It is interesting to note that at least in principle the proposed scheme may also be used to detect time-dilation effect that gravitational wave cause, in a setting similar to a non-mechanical gravitational wave detector proposed by Braginsky and Menskii [22,21]; fix two component quantum clocks on edges of a disk, making an angle $\pi/2$ with the origin of the disk. The disk is free-falling and is constantly rotating in phase with a frequency component of gravitational wave and the axis of rotation is pointing to the source of the wave. Then one clock's time is constantly dilated when compared with the other one's, due to gravitational field of the wave. However, it still seems to be a formidable task at present to detect gravitational wave using the proposed scheme.

In conclusion, we reported that the entangled pair of non-degenerate quantum clocks can be used as a specialized detector for precisely measuring the difference of proper-times that each constituent quantum clock experiences. We described why the proposed scheme would be more precise in the proper-time difference measurement than a scheme in which readings of two separate quantum clocks are compared. Acceleration's effects on quantum clock's time evolution may be non-negligible. In this case, we considered some experiments where the acceleration effect cancels with each other. The proposed scheme can be used in precision test of relativistic time dilation effects-the twin paradox effect [19], gravitation time-dilation [20,21], and possibly time-dilation due to the gravitational wave.

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References

1. A. Einstein, B. Podolsky, N. Rosen, Phys. Rev. **47**, 777 (1935); F. Selleri, *Quantum Mechanics versus Local Realism. The Einstein-Podolsky-Rosen Paradox* (Plenum, New York, 1988).
2. J.S. Bell, Physics **1**, 195 (1964), reprinted in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, 1987).
3. P. Shor, *Proc. 35th Ann. Symp. on Found. of Computer Science* (IEEE Comp. Soc. Press, Los Alomitos, CA, 1994), p. 124.
4. J.J. Bollinger, W.M. Itano, D.J. Wineland, Phys. Rev. A **54**, 4649 (1996).
5. A.N. Boto, P. Kok, D.S. Abrams, S.L. Braunstein, Phys. Rev. Lett. **85**, 2733 (2000).
6. M. D'Angelo, M.V. Chekhova, Y. Shih, Phys. Rev. Lett. **87**, 013602 (2001), `quant-ph/0103035` (available at `http://xxx.lanl.gov`).
7. R. Jozsa, D.S. Abrams, J.P. Dowling, C.P. Williams, Phys. Rev. Lett. **85**, 2010 (2000).
8. E.A. Burt, C.R. Ekstrom, T.B. Swanson, Phys. Rev. Lett. **87**, 129801 (2001), `quant-ph/0007030`.
9. M.S. Shahriar, `quant-ph/0010007`.
10. U. Yurtsever, J.P. Dowling, `quant-ph/0010097`.
11. J. Preskill, `quant-ph/0010098`.
12. I.L. Chuang, Phys. Rev. Lett. **85**, 2006 (2000).
13. V. Giovannetti, S. Lloyd, L. Maccone, Nature **412**, 417 (2001).
14. J. von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1955).
15. J. Preskill, Lecture notes on quantum information, available at `http://www.theory.caltech.edu/people/preskill/ph229/`
16. D. Bohm, B.J. Hiley, *The Undivided Universe* (Routledge, London and New York, 1993); P.R. Holland, *The Quantum Theory of Motion* (Cambridge University Press, 1993).
17. J.J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley, 1985).
18. V. Bužek, R. Derka, S. Massar, Phys. Rev. Lett. **82**, 2207 (1999).
19. E.F. Taylor, J.A. Wheeler, *Spacetime Physics* (W.H. Freeman and Company, San Francisco and London, 1966).
20. R.V. Pound, G.A. Rebka, Phys. Rev. Lett. **4**, 337 (1960).
21. C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation* (W.H. Freeman and Company, New York, 1973).
22. V.B. Braginsky, M.B. Menskii, Sov. Phys.-JETP Lett. **13**, 417 (1971).